Toward Model Based Dynamic Positioning of Underwater Robotic Vehicles

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Abstract— **This paper reports preliminary experimental evaluation of a family of model based trajectory-tracking controllers for the low-speed maneuvering of fully actuated underwater vehicles. First, previously reported studies are briefly reviewed. Second, one linear and five model-based based controllers are presented. Third, preliminary experimental studies are reported. The experimental results corroborate the analytical predictions that the model-based controllers outperform PD control over a wide range of operating conditions. The exactly linearizing model based controller is outperformed by its non exactly linearizing counterpart. The adaptive controllers are shown to provide reasonable online plant parameter estimates, and velocity and position tracking consistent with theoretical predictions — providing good velocity tracking and, with the appropriate parameter update law, position tracking. To the best of our knowledge, this is the first reported comparative experimental study of this class of model based controllers for underwater vehicles.**

I. Introduction

This paper addresses the trajectory tracking problem for the low-speed maneuvering of fully actuated underwater vehicles. It is organized as follows: First a brief review of previously reported control studies and plant models is presented. This is followed by the presentation of the controllers. Finally we report results from experimental trials using these controller.

The problem of trajectory tracking is defined as follows: Given the underwater vehicle dynamical plant model and a continuous bounded time varying reference trajectory, whose derivatives are continuous and bounded, design a control law that ensures asymptotically exact tracking of the reference trajectory. Finite dimensional approximations for the dynamics of underwater vehicles are structurally similar to the equations of motion for a fully-actuated rigid body — both classes of plants are nonlinear, yet the plant parameters enter linearly into the overall nonlinear differential equation of motion. For the case of second order systems with *linear* dynamics, classical linear controller design techniques apply directly. Early studies of trajectory tracking for rigid-body robot arms, employed linearized plant approximations in order to apply linear model-reference control techniques [27], [13], [44].

The controllers reported herein do not employ linearized plant approximations. Their structure is motivated by previously reported model-based approaches to the problem of trajectory tracking for fully nonlinear holonomic mechanical systems — robot arms. Most previous studies on robot arms have taken one of the three following general approaches:

1. **Linear Control — PD and PID:** The globally asymptotic regulation of a fully nonlinear mechanical system was first addressed for PD control of robot arms in [4], in which Lasalle's invariance theorem, [33], is employed to show that PD control provides globally asymptotic setpoint regulation.
The application of this approach to the control of underwater vehicles is the PD controller outlined in Section III-A.
2. **Nonlinear Model-Based Exact Linearization:** The case of exact, model based trajectory tracking control of robot arms was first reported independently in [36], [35] and [18], which report controllers with model-based feedforward terms that exactly linearize (without any approximation) the plant and result in globally asymptotically stable linear error dynamics. Working implementations of this approach were first reported in [1]. An adaptive version of this exact linearization approach was reported in [10], [9] and shown to be globally asymptotically stable in tracking error and locally stable in plant parameter error.
The application of this approach to the control of underwater vehicles is the *exact linearizing* controller (EL) outlined in Section III-B, and the *adaptive exact linearizing* controller (AEL) outlined in Section III-D.
3. **Nonlinear Model-Based Without Linearization:** The case of exact model based tracking control of robot arms *without exact linearization* was first reported in [39], [40] along with an adaptive extension that is globally asymptotically stable in position and velocity tracking error and globally stable in plant parameter error. Variations of this general approach are reported in [38], [47].
The application of this approach to the control of underwater vehicles is the *nonlinear* controller (NL) outlined in Section III-C, the *adaptive nonlinear* controller (ANL) outlined in Section III-E, and the *epsilon adaptive nonlinear* controller (ϵANL) outlined in Section III-F.

These ideas have each been applied, in various forms, to the problem of the control of underwater vehicles. A significant number of previously reported studies of the control of underwater vehicles employed linearized plant approximations both for the theoretically justified case of setpoint regulation as well as the theoretically unjustified case of trajectory tracking. In [20] the authors report a linear discrete-time approximation for underwater vehicle dynamics, and report simulations of linear quadratic and robust control methods for this class of discrete time linear plants. In [21], [31] the authors report linear quadratic control and self tuning control of a linearized vehicle dynamical plant model and numerical simulations. In [12],

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[28] the authors report Proportional-Integral (PI) and linear model-reference adaptive control of a linearized underwater vehicle plant model and experimental demonstrations. In [23], [25] the authors report the model-reference adaptive control of a linearized underwater vehicle dynamical plant model and numerical simulations. In [11] the authors report the sliding-mode control of a linearized underwater vehicle dynamical plant model and numerical simulations. A few studies, e.g. [29], have reported neural network and/or fuzzy logic control techniques for underwater vehicles and numerical simulations. Several reported studies address the special case of propulsion-less and appendage-less symmetric vehicles in inviscid fluid for which the PDE fluid component has a closed form solution, e.g. [34].

Relatively few control studies directly address decoupled nonlinear plant models for underwater vehicles. In [49] the authors report nonlinear sliding mode control for an underwater vehicle, and numerical simulations for X, Y, and heading. The first reported experiments of nonlinear adaptive model based control of an underwater vehicle are reported in [48]. In [6] the authors address the problem of model-based underwater vehicle state estimation. In [8], [50] the authors report an adaptive control approach to underwater vehicle control employing unbounded gains, and demonstrate the approach with experimental data.

Several studies address fully coupled nonlinear models for underwater vehicles, associated controllers, and report simulation studies, e.g. [16], [17], [3]. These approaches make explicit assumptions on the structure of the approximate finite-dimensional vehicle plant dynamics to ensure that the vehicle dynamical plant model possesses passivity properties identical to those possessed by rigid-body holonomic mechanical systems — an assumption that to the best of our knowledge has not been empirically validated.

A. Review of Dynamical Modelling of Underwater Vehicles

Exact analysis of a rigid-body underwater vehicle's dynamics includes both the finite-dimensional dynamic of the vehicle body itself and the infinite-dimensional dynamics of the fluid surrounding the vehicle. The former is a finite-dimensional dynamical system represented by an ordinary differential equation (ODE), but the latter is a continuous (infinite dimensional) dynamical system represented by the incompressible Navier-Stokes equation, a partial differential equation (PDE). Except for a few idealized cases of little practical utility (e.g. symmetric bodies in inviscid fluid) for which the PDE fluid component has a closed form solution, the numerical solution of the full vehicle (ODE) and fluid (PDE) dynamical system remains a formidable computational obstacle, and an area of active research [32].

The overwhelming computational complexity of PDE dynamical models has motivated the widespread use (by naval architects and others) of finite dimensional *approximate* models for marine vehicles, sometimes termed "lumped parameter models". The expedient of experimentally determining approximate finite-dimensional models for complex fluid phenomenon is widely employed in naval architecture. The most commonly accepted finite-dimensional dynamics models for *submarine vehicles* trace their lineage to studies performed at the U.S. Navy's David Taylor Model Basin beginning in the 1950's [22], [19] with subsequent revisions reported in [14], [15]. These second order nonlinear ODE dynamical models (known as "the DTRC standard submarine equations of motion") and subsequent enhancements have been adopted for use in the design of control systems for underwater robotic vehicles, in either linearized form, e.g. [23], or in full nonlinear form, e.g. [25].

Most reported finite dimensional plant models for holonomic (fully actuated) underwater vehicles take the following general form

$$\tau(t) = M(x_w(t), v(t))\dot{v}(t) + d(x_w(t), v(t)) + b(x_w(t)), \quad (1)$$

where $x_w(t) \in \mathbb{R}^{6x1}$ is a vector of the vehicle position and orientation in world inertial coordinates; $v(t) \in \mathbb{R}^{6x1}$ and $\dot{v}(t) \in \mathbb{R}^{6x1}$ are, respectively, vectors of the vehicle velocity and acceleration in vehicle body coordinates; $\tau(t) \in \mathbb{R}^{6x1}$ is a vector of control forces from thrusters and control surfaces in body coordinates; $M(x_w(t), v(t))$ is a mass matrix representing both rigid-body mass and velocity-dependent "added mass"; $d(x_w(t), v(t))$ is a vector of vehicle drag and coriolis forces; and $b(x_w(t))$ is a vector of vehicle buoyancy forces. Note that the velocity in inertial (world) coordinates $v_w(t)$, is related to the velocity in body coordinates by a linear transformation of the form $v_w(t) = T(x_w(t))v(t)$ [19]. The plant (1) can also be rewritten as

$$\begin{aligned} \dot{v}(t) &= M(x_w(t), v(t))^{-1}\tau - \\ &\quad M(x_w(t), v(t))^{-1}d(x_w(t), v(t)) - \\ &\quad M(x_w(t), v(t))^{-1}b(x_w(t)). \end{aligned} \quad (2)$$

At present, there is no uniform consensus within the research community on the exact analytical form of the terms comprising $M(x_w(t), v(t))$, $d(x_w(t), v(t))$, and $b(x_w(t))$, nor for the force vector $\tau(t)$. The few studies that have reported specific instances of (1), e.g. [25], have based their component terms on the DTRC standard submarine equations of motion [19], [15].

Although not theoretically justified, in this report we adopt the common practice of further approximating the full six degree of freedom equations by neglecting off diagonal entries and coupling terms, coriolis forces, tether dynamics, as well as assuming a constant added mass [37], [7]. The resulting decoupled single degree of freedom dynamical equations take the form

$$\begin{aligned} \tau_i(t) &= m_i\dot{v}_i(t) + d_{Q_i}v_i(t)|v_i(t)| + d_{L_i}v_i(t) + b_i, \\ &\quad m_i > 0;\ d_{L_i}, d_{Q_i} > 0, \end{aligned} \quad (3)$$

or, rewritten,

$$\dot{v}_i(t) = m_i^{-1}\tau_i(t) - m_i^{-1}d_{Q_i}v_i(t)|v_i(t)| - \quad (4)$$

TABLE I

NOMENCLATURE

Degree of Freedom	Force Moment	Linear Velocity Angular Velocity	Linear Position Angular Position
1: X Translation (Surge)	$\tau_1(t)$	$v_1(t)$	$x_1(t)$
2: Y Translation (Sway)	$\tau_2(t)$	$v_2(t)$	$x_2(t)$
3: Z Translation (Heave)	$\tau_3(t)$	$v_3(t)$	$x_3(t)$
4: Rotation About Z(Yaw/HDG)	$\tau_4(t)$	$v_4(t)$	$x_4(t)$
5: Rotation About Y(Pitch)	$\tau_5(t)$	$v_5(t)$	$x_5(t)$
6: Rotation About X(Roll)	$\tau_6(t)$	$v_6(t)$	$x_6(t)$

$$m_i^{-1} d_{L_i} v_i(t) - m_i^{-1} b_i,$$

where, for each degree of freedom i, $\tau_i(t)$ is the net control force, m_i is the effective mass, $d_{Q_i} v_i(t)|v_i(t)|$ and $d_{L_i} v_i(t)$ represent the hydrodynamic drag, and b_i is the buoyancy. Using lumped parameters, (4) can be written as:

$$\dot{v}_i(t) = \alpha_i \tau_i + \beta_i v_i(t)|v_i(t)| + \mu_i v_i(t) + \nu_i. \tag{5}$$

The decoupled single degree of freedom, lumped parameter, dynamical plant model can be written in vector form as

$$\dot{v}(t)_i = \Phi_i^T f(t)_i, \tag{6}$$

where $\Phi_i = [\alpha_i; \beta_i; \mu_i; \nu_i]_{4\times1}$ is the vector of lumped plant parameters and $f(t)_i = [\tau(t)_i; v_i(t)|v_i(t)|; v_i(t); 1]_{4\times1}$ is a nonlinear vector of state and the control input $\tau(t)_i$. The lumped parameters are defined in Table II.

TABLE II

LUMPED PARAMETERS

Lumped Parameters	Physical Definition
α_i	m_i^{-1}
β_i	$-m_i^{-1} d_{Q_i}$
μ_i	$-m_i^{-1} d_{L_i}$
ν_i	$-m_i^{-1} b_i$

B. Review of Dynamic Positioning of Underwater Vehicles

Most dynamically positioned marine vehicles in use today employ Proportional-Derivative (PD) or Proportional-Integral-Derivative (PID) controllers for each controlled degree-of-freedom of the form

$$\tau_d \quad = K_p \Delta x_{body} + K_d \Delta v + K_i \int_0^t \Delta x_{body}(\tau) d\tau \tag{7}$$

where Δx_{body} and Δv are the position and velocity tracking errors, respectively, in body coordinates, and K_p, K_i, and K_d are empirically tuned feedback gain matrices, and τ_d is the vector of net forces and moments commanded to the vehicle thruster system.

The development of model-based control techniques for the dynamic positioning of underwater robotic vehicles has been limited by the paucity of *experimentally validated* plant models. Although a variety of authors have reported the development dynamical models for underwater vehicles, very few report direct experimental validation of their reported plant dynamics models for low-speed maneuvering. Most control studies demonstrate the performance of the resulting closed-loop system in computer simulations alone, e.g. [21], [11], [45], [2], or employ non-parametric control methodologies that do not require knowledge of the plant dynamics [8]. Relatively few studies, e.g. [24], [48], have reported experiments which experimentally identify the plant parameters for dynamical plant models in multiple degrees of freedom; even fewer studies, [37], [7], [42], report both experimentally determined model parameters *and* compare experimentally observed vehicle dynamical response with that predicted by the proposed analytical models.

II. Decoupled Single Degree of Freedom Dynamical Plant Model for the JHU ROV

In this section we present experimentally identified, decoupled, single degree of freedom dynamical plant models of the form (4), for the Johns Hopkins University (JHU) Remotely Operated Vehicle (ROV). A set of plant model parameters was identified for the x_1, x_2, x_3 , and x_4 degrees of freedom using a stable adaptive identifier, originally reported in [42]. The scalar adaptive identifier was run on experimental data, collected by conducting dynamic vehicle trials in which the JHU ROV was commanded to follow an open loop sinusoidal thrust profile. The output of this process is an estimated plant velocity, $\hat{v}(t)_i$, as well as estimates for the unknown plant parameters.

How well does the identified plant model performance agree with experimentally observed actual vehicle motion? To answer this question we ran simulations on the adaptively identified plant parameters, in each degree of freedom i. The output of the simulations was a velocity, v_{model_i}. The logged experimental plant velocity, v_{p_i}, was then compared to the velocity predicted by the dynamical plant model, v_{model_i}. The error for each degree of freedom i, is calculated as $e_i = mean(|v_{model_i} - v_{p_i}|)$. The best identified parameters were those with the lowest "error", and are listed in Table III.

As seen by the error listed in Table III, the identified dynamical plant models were able to predict the dynamical behavior of the JHU ROV down to $1.6\, cm/s$ in the x_1, x_2, and x_3 DOF, and $1.651\, degrees/s$ in the x_4 DOF.

III. Model Based Control

In this section we present five different model based controllers and a basic Proportional-Derivative (PD) con-

TABLE III
Adaptive ID Lumped Parameters

Degree of Freedom	α_i	β_i	μ_i	ν_i	Error e_i
1 (x_1)	0.003755	-4.406	0	0.007358	0.01664 m/s
1 (x_2)	0.002355	-4.9952	0	-0.004066	0.01607 m/s
3 (x_3)	6.237e-4	-1.102	0	0.01934	0.01652 m/s
4 (x_4)	0.0102	-1.9086	0	-0.001221	1.651 degrees/s
Degree of Freedom	Inertia	D_Q	D_L	Buoyancy	
1 (x_1)	266.3 [kg]	1173.4 [kg/m]	0 [kg/s]	-1.96 [N]	
2 (x_2)	424.6 [kg]	2120.9 [kg/m]	0 [kg/s]	1.73 [N]	
3 (x_3)	1603.3 [kg]	1766.84 [kg/m]	0 [kg/s]	-31.01 [N]	
4 (x_4)	98.04 [kg m^2]	187.1 [kg m^2]	0 [(kg m^2)/s]	0.1197 [N-m]	

troller. The stability of these controllers and their simulated performance is reported in [43].

A. PD Controller

The basic PD controller takes the form

$$\begin{aligned}\tau(t) &= k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad k_p, k_d > 0,\end{aligned} \tag{8}$$

where k_p and k_d are scalar error feedback gains. The state error coordinates are defined as

$$\begin{aligned}\Delta x(t) &= x_d(t) - x(t),\\ \Delta v(t) &= v_d(t) - v(t),\\ \Delta\dot{v}(t) &= \dot{v}_d(t) - \dot{v}(t),\end{aligned} \tag{9}$$

where $\dot{v}_d(t)$, $v_d(t)$, and $x_d(t)$ are the desired acceleration, velocity, and position. Note that $v(t) = d(x(t))/dt$. Substituting (8) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}&m\dot{v}(t) + d_Q v(t)|v(t)| + d_L v(t) + b-\\ &\quad k_p\Delta x(t) - k_d\Delta v(t) = 0.\end{aligned} \tag{10}$$

In the case where the buoyancy term, b, is zero, the PD controller will perform setpoint regulation but not trajectory tracking.

B. Exact Linearizing Model Based Controller

The Exact Linearizing Model Based Controller (EL) takes the form

$$\begin{aligned}\tau(t) &= m\dot{v}_d(t) + d_Q v(t)|v(t)| +\\ &\quad d_L v_d(t) + b + k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad m, k_p, k_d > 0,\ d_Q, d_L \geq 0,\end{aligned} \tag{11}$$

where m, d_Q, d_L, and b are known scalar quantities, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The state error coordinates are defined in (9). Substituting (11) into (3) the resulting closed loop dynamical system is

$$m\Delta\dot{v}(t) + (d_L + k_d)\Delta v(t) + k_p\Delta x(t) = 0, \tag{12}$$

a time varying linear system.

The EL fixed model based controller provides asymptotically exact position and velocity trajectory tracking. In addition all signals remain bounded, as reported in [43].

C. Non-Linear Model Based Controller

The Non-Linear Model Based Controller (NL) takes the form

$$\begin{aligned}\tau(t) &= m\dot{v}_d(t) + d_Q v_d(t)|v_d(t)| + d_L v_d(t) +\\ &\quad b + k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad m, k_p, k_d > 0,\ d_Q, d_L \geq 0,\end{aligned} \tag{13}$$

where m, d_Q, d_L, and b are known scalar quantities, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The state error coordinates are defined in (9). Substituting (13) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}&m\Delta\dot{v}(t) + d_q(v_d(t)|v_d(t)| - v(t)|v(t)|)+\\ &\quad (d_L + k_d)\Delta v(t) + k_p\Delta x(t) = 0.\end{aligned} \tag{14}$$

or rewritten as

$$\begin{aligned}&\Delta\dot{v}(t) = -m^{-1}(d_q(v_d(t)|v_d(t)| - v(t)|v(t)|)+\\ &\quad (d_L + k_d)\Delta v(t) + k_p\Delta x(t)).\end{aligned} \tag{15}$$

The NL fixed model based controller provides stable position tracking and asymptotically exact velocity tracking. Additionally, all signals remain bounded, as reported in [43].

D. Adaptive Exact Linearizing Model Based Controller

The Adaptive Exact Linearizing Model Based Controller (AEL) takes the form

$$\begin{aligned}\tau(t) &= \hat{m}(t)\dot{v}_d(t) + \hat{d}_Q(t)v(t)|v(t)| +\\ &\quad \hat{d}_L(t)v_d(t) + \hat{b}(t) + k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad k_p, k_d > 0,\end{aligned} \tag{16}$$

where $\hat{m}(t)$, $\hat{d}_Q(t)$, $\hat{d}_L(t)$, and $\hat{b}(t)$ are adaptive estimates of the scalar plant parameter values, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The plant parameter error coordinates are defined as

$$\begin{aligned}\Delta m(t) &= \hat{m}(t) - m,\\ \Delta d_Q(t) &= \hat{d}_Q(t) - d_Q,\\ \Delta d_L(t) &= \hat{d}_L(t) - d_L,\\ \Delta b(t) &= \hat{b}(t) - b,\\ \Delta\Psi(t) &= \hat{\Psi}(t) - \Psi,\\ \Delta\dot{\Psi}(t) &= \dot{\hat{\Psi}}(t);\ \dot{\Psi} = 0,\end{aligned} \tag{17}$$

where $\hat{\Psi}(t) = [\hat{m}(t); \hat{d}_Q(t); \hat{d}_L(t); \hat{b}(t)]_{4\times 1}$ is an estimate of the unknown plant parameter vector $\Psi = [m; d_Q; d_L; b]_{4\times 1}$. Substituting (16) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}\Delta m(t)\dot{v}_d(t) + \Delta d_Q(t)v(t)|v(t)| + \Delta d_L(t)v_d(t) +\\ \Delta b(t) + m\Delta\dot{v}(t) + d_L\Delta v(t) +\\ k_p\Delta x(t) + k_d\Delta v(t) = 0,\end{aligned} \tag{18}$$

or rewritten as

$$\begin{aligned}\Delta\dot{v}(t) = -m^{-1}(\Delta\Psi(t)^T X(t) +\\ (d_L + k_d)\Delta v(t) + k_p\Delta x(t)),\end{aligned} \tag{19}$$

where $X(t) = [\dot{v}_d(t); v(t)|v(t)|; v_d(t); 1]_{4\times 1}$. Note that if the parameter error $\Delta\Psi(t)$, were zero, then (18) becomes the EL closed loop dynamical system (12).

All signals remain bounded using the AEL controller. Further, asymptotically exact velocity tracking, and the asymptotic convergence of the time derivative of the adaptive parameter estimates to zero is reported for the AEL controller in [43].

E. Adaptive Non-Linear Model Based Controller

In this section the Adaptive Non-Linear Model Based Controller is presented. The ANL takes the form:

$$\begin{aligned}\tau(t) &= \hat{m}(t)\dot{v}_d(t) + \hat{d}_Q(t)v_d(t)|v_d(t)| +\\ &\quad \hat{d}_L(t)v_d(t) + \hat{b}(t) + k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad k_p, k_d > 0,\end{aligned} \tag{20}$$

where $\hat{m}(t)$, $\hat{d}_Q(t)$, $\hat{d}_L(t)$, and $\hat{b}(t)$ are estimates of the scalar plant parameter values, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The plant parameter error coordinates are defined in (17) and the state error coordinates are defined in (9). Substituting (20) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}\Delta m(t)\dot{v}_d(t) + \Delta d_Q(t)v_d(t)|v_d(t)| +\\ \Delta d_L(t)v_d(t) + \Delta b(t) + m\Delta\dot{v}(t) +\\ d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|) + d_L\Delta v(t) +\\ k_p\Delta x(t) + k_d\Delta v(t) = 0,\end{aligned} \tag{21}$$

which can be rewritten as

$$\begin{aligned}\Delta\dot{v}(t) = -m^{-1}(\Delta\Psi(t)^T X(t) +\\ d_q(v_d(t)|v_d(t)| - v(t)|v(t)|) +\\ (d_L + k_d)\Delta v(t) + k_p\Delta x(t)),\end{aligned} \tag{22}$$

where $X(t) = [\dot{v}_d(t); v_d(t)|v_d(t)|; v_d(t); 1]_{4\times 1}$. Note that if the parameter error were zero, $\Delta\Psi(t) = 0$, then (21) becomes the NL closed loop dynamical system (14).

The ANL controller provides asymptotically exact velocity tracking and stable position tracking. Further, the time derivative of the adaptive parameter estimates asymptotically converges to zero and all signals remain bounded [43].

F. Epsilon Adaptive Model Based Controller

Although the ANL controller was shown to be stable, bounded, and to provide asymptotically exact velocity tracking, it does not guarantee asymptotically exact position tracking. This apparent defect can be corrected with a slightly modification of the parameter update law. The idea is to introduce a small cross term of the form $\epsilon\Delta x(t)\Delta v(t)$; $\epsilon > 0$, to the Lyapunov function. If ϵ is sufficiently small, the Lyapunov function remains positive definite and radially unbounded and its time derivative is locally negative definite. This local stability result was first reported independently in [4], [30], [46]. A globally stable variation was reported in [47]. Applying this approach to the present plant, (3), the control law (20) remains unchanged, resulting in the closed loop dynamical system (21).

The resulting parameter update law is

$$\dot{\hat{\Psi}}(t) = \Gamma X(t)(\Delta v(t) + \epsilon\Delta x(t)), \tag{23}$$

where $X(t) = [\dot{v}_d(t), v_d(t)|v_d(t)|, v_d(t), 1]_{4\times 1}$, $\epsilon > 0$, $\gamma_i \geq 0$, and $\Gamma_{4\times 4} = diag[\gamma_i]$ for $i = 1\cdots 4$.

This controller can be shown to be locally asymptotically stable in position and velocity tracking error, and stable in plant parameter error [43].

IV. Experimental Setup

The JHU ROV, Fig. 1, is a tethered, remotely operated underwater robot. The JHU ROV is powered by an isolated 10kW DC power supply. The mass of the vehicle is 140 kg and it's dimensions are 1.5m long × 1m wide × 0.6m high. The position and heading of the vehicle are actively controlled and the vehicle is passively stable in roll and pitch. Actuation is provided by five DC brushless, electric thrusters. A complete description of the JHU ROV is reported in [41].

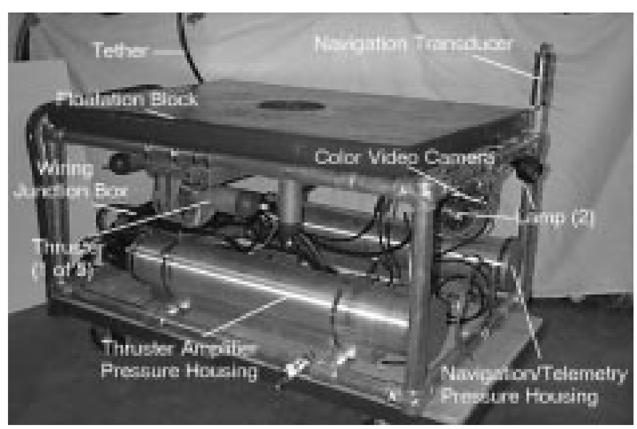

Fig. 1. Physical Layout of the JHU ROV

The ROV is equipped for full 6-DOF position measurement. For this set of experiments, XYZ position was measured using a 300 kHz Sonic High Accuracy Ranging and Positioning System (SHARPS) time-of-flight acoustic

TABLE IV

INSTRUMENTATION

Variable	Sensor	Precision	Update Rate
XYZ Position	300 kHz SHARPS Acoustic Transponder System	0.5 cm	10 Hz
Depth	Paroscientific	10ppm FS (0.007m)	8 Hz
Heading	Litton LN200 IMU Gyro (USNA)	0.01 deg.	20 Hz
Roll and Pitch	KVH ADGC	0.1 deg.	20 Hz
Heading	KVH ADGC	1 deg	20 Hz

hardwired transponder system. Depth was instrumented via a Paroscientific 8DP-700-1 Digiquartz depth sensor, sampled at 8 Hz. A KVH Azimuth Digital Gyro Compass (ADGC) measured the vehicle's roll and pitch. Heading was measured using a prototype Litton LN200 IMU. The vehicle's entire sensor suite is listed in Table IV.

Free running experiments were conducted in the x_1, x_2, x_3, and x_4 degrees of freedom (DOF) to identify the plant model parameters. Multiple trials in each DOF were conducted with sinusoidal thrust profiles of varying peak magnitude and frequency. A static thrust model was used for these experiments to model the thrust produced by each thruster — i.e. the thrust, τ, produced is assumed to be proportional to the current commanded, i_c, as in $\tau = k_t i_c$. Current was controlled using current mode amplifiers. The authors are aware of more advanced thruster models [26], [5] that are more precise, however they require a dynamical thruster characterization not available at the time of the experiments.

Free running closed loop experiments were conducted in the x_1, x_2, x_3,and x_4 DOF to test the controllers. Multiple trials were conducted using each controller tracking sinusoidal position and velocity reference trajectories of varying peak magnitude and frequency. The velocity trajectory is the time derivative of the position trajectory. Wherever position is specified in this paper, it is in inertial world coordinates, whereas velocity is in body coordinates. Note that the velocity in inertial (world) coordinates $v_w(t)$, is related to the velocity in body coordinates by a linear transformation of the form $v_w(t) = T(x_w(t))v(t)$ [19]. Whenever specifications such as peak magnitude and frequency are given in this paper regarding reference trajectories, they are for a position reference trajectory.

The experiments reported in this paper were conducted using the JHU ROV at the United States Naval Academy's Hydromechanics Laboratory, in Annapolis, MD. The work was done in collaboration with Dr. Dan Stilwell, of the United States Naval Academy, who deployed his IMU on the JHU vehicle during these experiments to examine IMU calibration.

V. Experimental Results — Comparative Performance

A direct comparison of the performance of the EL, NL, and PD controllers while tracking a sinusoidal trajectory of the same peak magnitude and frequency will be presented first. This is followed by a comparison of the performance of the adaptive model based controllers (AEL, ANL, and ϵANL) to the fixed model based controllers (EL and NL) tracking the same trajectory.

A. PD vs Model Based Control

The PD, EL, and NL controllers were implemented to track a sinusoidal reference trajectory of the same peak magnitude and frequency. For each degree of freedom, each controller used the same set of PD gains, k_p and k_d, listed in Table V. The position error re-

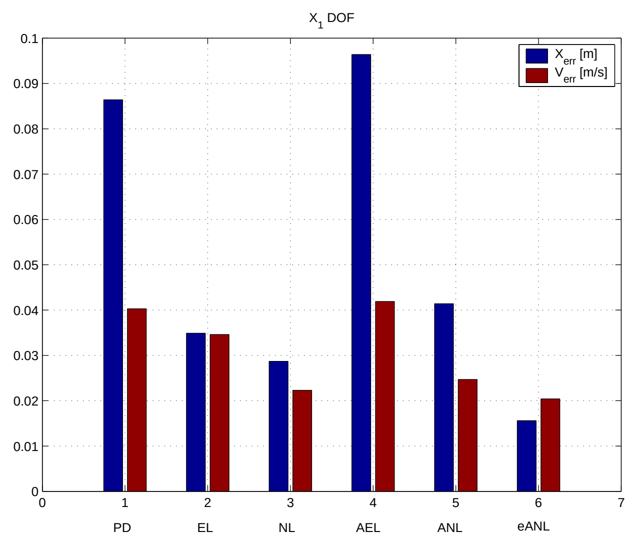

Fig. 2. Plot of tracking error for the PD, EL, NL, AEL, ANL, and ϵANL controllers in the x_1 DOF. Reference trajectory is a sinusoid of peak magnitude 0.5 m and frequency 0.3925 rad/s.

TABLE V

PD VS MODEL BASED COMPARISON

DOF	Magnitude	Frequency	k_p	k_d
x_1	0.5 m	0.3925 rad/s	500	200
x_2	0.25 m	0.3925 rad/s	500	200
x_3	0.2 m	0.524 rad/s	600	800
x_4	45 degrees	0.3925 rad/s	250	100

ported for each degree of freedom i, was calculated as $X_{err_i} = mean(|x_{d_i} - x_{act_i}|)$ where x_{d_i} and x_{act_i} are the desired/reference and actual logged position of the JHU ROV in degree of freedom i. The velocity error was calculated as $V_{err_i} = mean(|v_{d_i} - v_{act_i}|)$, where v_{d_i} and v_{act_i} are the desired/reference and actual logged velocity of the JHU ROV in DOF i. Bar charts displaying both the position and velocity tracking error for each controller in each degree of freedom can be seen in Fig. 2, Fig. 3, Fig. 4, and Fig. 5. Based on the error metric reported,

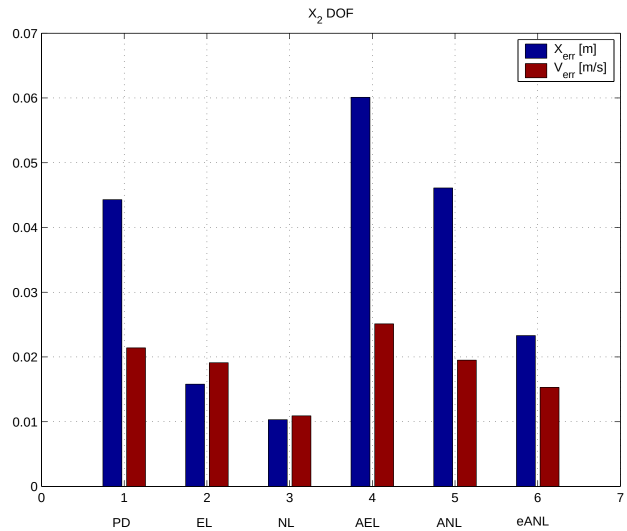

Fig. 3. Plot of tracking error for the PD, EL, NL, AEL, ANL, and ϵANL controllers in the x_2 DOF. Reference trajectory is a sinusoid of peak magnitude 0.25 m and frequency 0.3925 rad/s.

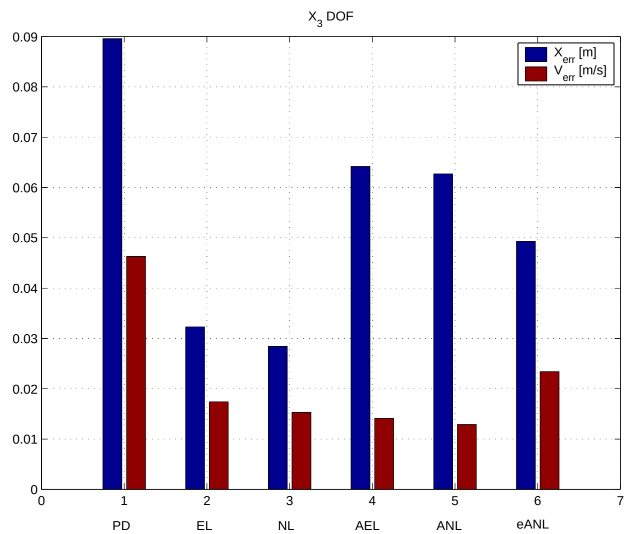

Fig. 4. Plot of tracking error for the PD, EL, NL, AEL, ANL, and ϵANL controllers in the x_3 DOF. Reference trajectory is a sinusoid of peak magnitude 0.2 m and frequency 0.524 rad/s.

it is clear that (a), the NL controller performs as well or better than the EL controller in each DOF, and (b), both fixed model based controllers outperform the basic PD controller tracking the trajectory.

B. Adaptive vs Fixed Model Based Control

The adaptive model based controllers were run on the same trajectories as the non-adaptive controllers using the same sets of PD gains, lister in Table V. The adaptive controllers have the added complexity of setting the adaptation gains, γ_i. For the purpose of this direct comparison, the adaptation gains were the same for the AEL, ANL, and the ϵANL controllers in each degree of freedom. The adaptation gains can be found in Table VI. Comparing the performance of the AEL, ANL, and ϵANL controllers,

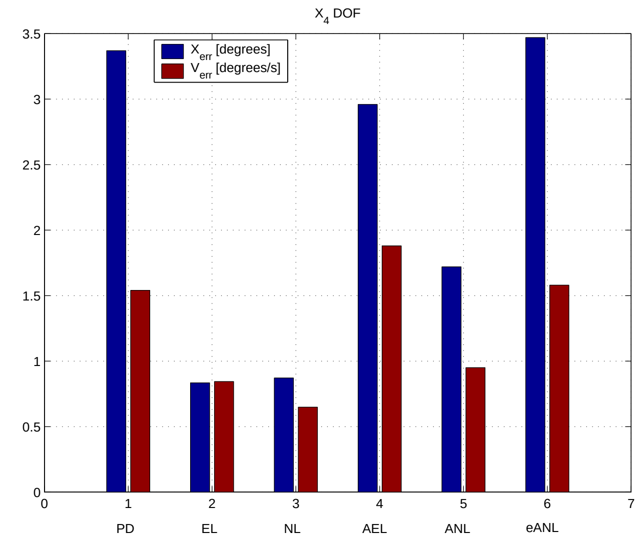

Fig. 5. Plot of tracking error for the PD, EL, NL, AEL, ANL, and ϵANL controllers in the x_4 DOF. Reference trajectory is a sinusoid of peak magnitude 45 degrees and frequency 0.3925 rad/s.

TABLE VI
ADAPTATION GAINS

DOF	γ_1	γ_2	γ_3	γ_4
x_1	5000	100000	0	0.01
x_2	5000	100000	0	0.01
x_3	50000	5000000	0	1
x_4	500	50000	0	1

the ϵANL controller did the best job tracking the position trajectory in all but the x_4 DOF. The ϵANL controller also tracked the velocity reference trajectory the best in the x_1 and x_2 DOF. The ANL controller performed the best of the adaptive controllers in tracking the velocity in the x_3 and x_4 DOF.

If we compare the performance of the AEL to the fixed model based controllers, we see that it did a worse job in tracking both the position and velocity trajectories than did the fixed model based controllers. Similarly, the ANL did not perform as well as either of the fixed model based controllers. However, when compared directly to the basic PD controller, we see that the ANL outperformed the basic PD controller in the x_1, x_3, and x_4 degree of freedom, and performed comparably to the basic PD controller in the x_2 degree of freedom. The AEL also outperformed the basic PD controller in the x_3 and x_4 degree of freedom, but fared worse when used to track a position and velocity trajectory in the x_1 and x_2 degree of freedom. The ϵANL controller performed as well as or better than the fixed model based controllers and clearly outperformed the basic PD controller in all but the x_4 degree of freedom.

We conclude that all the model based controllers (EL, NL, AEL, ANL, and ϵANL) outperform the basic PD controller in trajectory tracking. In comparing the fixed with the adaptive model based controllers, we conclude that if the right adaptation law is chosen, the adaptive model based controllers can perform better than the fixed model based controllers. By this we mean that while the AEL and ANL controllers consistently performed worse than the NL and EL fixed model based controllers, the ϵANL controller in general performed better or comparably to the El and NL controllers.

VI. Conclusion

This paper has reported a preliminary experimental evaluation of a family of model based trajectory tracking controllers for the low-speed maneuvering of fully actuated underwater vehicles.

We considered the simple, but empirically validated, case of a decoupled plant dynamics model employing constant added mass, linear and quadratic drag, buoyancy, and thrust input. We compared the performance of PD control, two fixed model-based controllers: an exactly linearizing controller (EL), a nonlinear controller (NL) that does not exactly linearize, and the performance of the adaptive extensions of the fixed model based controllers. The experiments of the closed-loop performance of these systems corroborate the theoretical predictions. The fixed model-based controllers outperformed the PD controller in trajectory tracking. The NL controller outperforms the exact-linearizing EL controller in trajectory tracking. We surmise that this performance difference is due to the fact that the EL controller attempts to exactly cancel nonlinear plant drag terms relying on the plant model parameters to be *exactly* correct, whereas the NL controller exploits the stabilizing property of the vehicle's natural quadratic damping. The adaptive model-based controllers all provide accurate velocity tracking, and the parameter estimates remain bounded and converge to fixed values. The addition of a position error feedback term to the adaptation update law in the ϵANL adaptive controller provides for accurate position tracking.

The experiments suggest that the model based controllers are robust with respect to (a) sensor noise and inaccuracy; (b) thruster control accuracy; and (c) unmodeled plant dynamics.

The experiments do not reveal the effects of variations in feedback gains, adaptation gains, fixed controller model parameter matching, or variations in trajectory profiles. We hope to shortly report an experimental evaluation of these conditions on the performance of these controller on actual underwater vehicles.

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