Toward Model Based Trajectory Tracking of Underwater Robotic Vehicles: Theory and Simulation

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Abstract

This paper addresses the trajectory tracking problem for the low-speed maneuvering of fully actuated underwater vehicles. First, previously reported studies are briefly reviewed followed by a review of a simple decoupled dynamical plant model. Second, one linear and five model-based controllers are presented. Their stability is evaluated analytically. Third, simulation studies are reported. The simulations corroborate the analytical predictions that model-based controllers provide superior tracking performance in comparison to Proportional-Derivative control

1 Introduction

This paper addresses the trajectory tracking problem for the low-speed maneuvering of fully actuated underwater vehicles. It is organized as follows: first a brief review of previously reported studies is presented. This is followed by the presentation of the controllers and analytical evaluation of their stability. Finally we report results from numerical simulations using these controller.

The problem of trajectory tracking is defined as follows: Given the underwater vehicle dynamical plant model and a continuous bounded time varying reference trajectory, whose derivatives are continuous and bounded, design a thrust control law that ensures asymptotically exact tracking of the reference trajectory. Finite dimensional approximations for the dynamics of underwater vehicles are structurally similar to the equations of motion for a fully-actuated rigid body — both classes of plants are nonlinear, yet the plant parameters enter linearly into the overall nonlinear differential equation of motion. For the case of second order systems with *linear* dynamics, classical linear controller design techniques apply directly. Early studies of trajectory tracking for rigid-body robot arms, employed linearized plant approximations in order to apply linear model-reference control techniques [25, 12, 39].

The controllers reported herein do not employ linearized plant approximations. Their structure is motivated by previously reported model-based approaches to the problem of trajectory tracking for fully nonlinear holonomic mechanical systems — robot arms. Most previous studies on robot arms have taken one of the three following general approaches:

1. **Linear Control - PD and PID:** The globally asymptotic regulation of a fully nonlinear mechanical system was first addressed for PD control of robot arms in [4], in which Lasalle's Invariance Theorem, [30], is employed to show that PD control provides globally asymptotic setpoint regulation.

   The application of this approach to the control of underwater vehicles is the PD controller outlined in Section 3.1.

2. **Nonlinear Model-Based Exact Linearization:** The case of exact, model based trajectory tracking control of robot arms was first reported independently in [34, 32] and [17], which report controllers with model-based feedforward terms that exactly linearize (without any approximation) the plant and result in globally asymptotically stable linear error dynamics. Working implementations of this approach were first reported in [1]. An adaptive version of this exact linearization approach

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was reported in [9, 8] and shown to be globally asymptotically stable in tracking error and locally stable in plant parameter error.

The application of this approach to the control of underwater vehicles is the *exact linearizing* controller (EL) outlined in Section 3.2, and the *adaptive exact linearizing* controller (AEL) outlined in Section 3.4.

3. **Nonlinear Model-Based Without Linearization:** The case of exact model based tracking control of robot arms *without exact linearization* was first reported in [36, 37] along with an adaptive extension that is globally asymptotically stable in position and velocity tracking error and globally stable in plant parameter error. Variations of this general approach are reported in [35, 42].

   The application of this approach to the control of underwater vehicles is the *nonlinear* controller (NL) outlined in Section 3.3, the *adaptive nonlinear* controller (ANL) outlined in Section 3.5, and the *epsilon adaptive nonlinear* controller (ϵANL) outlined in Section 3.6.

These ideas have each been applied, in various forms, to the problem of the control of underwater vehicles. A significant number of previously reported studies of the control of underwater vehicles employed linearized plant approximations both for the theoretically justified case of setpoint regulation as well as the case of trajectory tracking. In [19] the authors report a linear discrete-time approximation for underwater vehicle dynamics, and report simulations of linear quadratic and robust control methods for this class of discrete time linear plants. In [20, 29] the authors report linear quadratic control and self tuning control of a linearized vehicle dynamical plant model and numerical simulations. In [11, 26] the authors report Proportional-Integral (PI) and linear model-reference adaptive control of a linearized underwater vehicle plant model and experimental demonstrations. In [22, 24] the authors report the model-reference adaptive control of a linearized underwater vehicle dynamical plant model and numerical simulations. In [10] the authors report the sliding-mode control of a linearized underwater vehicle dynamical plant model and numerical simulations. A few studies, e.g. [27], have reported neural network and/or fuzzy logic control techniques for underwater vehicles and numerical simulations.

Relatively few control studies directly address decoupled nonlinear plant models for underwater vehicles. In [44] the authors report nonlinear sliding mode control for an underwater vehicle, and numerical simulations for X, Y, and heading. The first reported experiments of nonlinear adaptive model based control of an underwater vehicle are reported in [43]. In [5] the authors address the problem of model-based underwater vehicle state estimation. In [7, 45] the authors report an adaptive control approach to underwater vehicle control employing unbounded gains, and demonstrate the approach with experimental data.

Several studies address fully coupled nonlinear models for underwater vehicles, associated controllers, and report simulation studies, e.g. [15, 16, 3]. These approaches make explicit assumptions on the structure of the approximate finite-dimensional vehicle plant dynamics to ensure that the vehicle dynamical plant model possesses passivity properties identical to those possessed by rigid-body holonomic mechanical systems — an assumption that to the best of our knowledge has not been empirically validated.

1.1 Review of Dynamical Modelling of Underwater Vehicles

The most commonly accepted finite-dimensional dynamics models for *submarine vehicles* trace their lineage to studies performed at the U.S. Navy's David Taylor Model Basin beginning in the 1950's [21, 18] with subsequent revisions reported in [13, 14]. These second order nonlinear ODE dynamical models (known as "the DTRC standard submarine equations of motion") and subsequent enhancements have been adopted for use in the design of control systems for underwater robotic vehicles, in either linearized form, e.g. [22], or in full nonlinear form, e.g. [24].

Most reported finite dimensional plant models for holonomic (fully actuated) underwater vehicles take the following general form

$$\tau(t) = M(x_w(t), v(t))\dot{v}(t) + d(x_w(t), v(t)) + b(x_w(t)), \tag{1}$$

where $x_w(t) \in \mathbb{R}^{6x1}$ is a vector of the vehicle position and orientation in world inertial coordinates; $v(t) \in \mathbb{R}^{6x1}$ and $\dot{v}(t) \in \mathbb{R}^{6x1}$ are, respectively, vectors of the vehicle velocity and acceleration in vehicle body coordinates; $\tau(t) \in \mathbb{R}^{6x1}$ is a vector of control forces from thrusters and control surfaces in body coordinates; $M(x_w(t), v(t))$ is a mass matrix representing both rigid-body mass and velocity-dependent "added mass"; $d(x_w(t), v(t))$ is a vector of vehicle drag and coriolis forces; and $b(x_w(t))$ is a vector of vehicle buoyancy forces. Note that the velocity in inertial (world) coordinates $v_w(t)$, is re-

Degree of Freedom	Force Moment	Linear Velocity Angular Velocity	Linear Position Angular Position
1: X Translation (Surge)	$\tau_1(t)$	$v_1(t)$	$x_1(t)$
2: Y Translation (Sway)	$\tau_2(t)$	$v_2(t)$	$x_2(t)$
3: Z Translation (Heave)	$\tau_3(t)$	$v_3(t)$	$x_3(t)$
4: Rotation About Z(Yaw/HDG)	$\tau_4(t)$	$v_4(t)$	$x_4(t)$
5: Rotation About Y(Pitch)	$\tau_5(t)$	$v_5(t)$	$x_5(t)$
6: Rotation About X(Roll)	$\tau_6(t)$	$v_6(t)$	$x_6(t)$

Table 1: Nomenclature

lated to the velocity in body coordinates by a linear transformation of the form $v_w(t) = T(x_w(t))v(t)$ [18]. The plant (1) can also be rewritten as

$$\begin{aligned}\dot{v}(t) \quad = \quad & M(x_w(t), v(t))^{-1}\tau - \\ & M(x_w(t), v(t))^{-1}d(x_w(t), v(t)) - \\ & M(x_w(t), v(t))^{-1}b(x_w(t)).\end{aligned} \quad (2)$$

At present, there is no uniform consensus within the research community on the exact analytical form of the terms comprising $M(x_w(t), v(t))$, $d(x_w(t), v(t))$, and $b(x_w(t))$, nor for the force vector $\tau(t)$. The few studies that have reported specific instances of (1), e.g. [24], have based their component terms on the DTRC standard submarine equations of motion [18, 14].

Although not theoretically justified, in this report we adopt the common practice of further approximating the 6-DOF equations by neglecting off diagonal entries and coupling terms, coriolis forces, tether dynamics, as well as assuming a constant added mass [33, 6]. The resulting decoupled single degree of freedom dynamical equations take the form

$$\begin{aligned}\tau_i(t) \quad = \quad & m_i\dot{v}_i(t) + d_{Q_i}v_i(t)|v_i(t)| + d_{L_i}v_i(t) + b_i, \\ & m_i > 0;\ d_{L_i}, d_{Q_i} > 0,\end{aligned} \quad (3)$$

or, rewritten,

$$\begin{aligned}\dot{v}_i(t) \quad = \quad & m_i^{-1}\tau_i(t) - m_i^{-1}d_{Q_i}v_i(t)|v_i(t)| - \\ & m_i^{-1}d_{L_i}v_i(t) - m_i^{-1}b_i,\end{aligned} \quad (4)$$

where, for each degree-of-freedom i, $\tau_i(t)$ is the net control force, m_i is the effective mass, $d_{Q_i}v_i(t)|v_i(t)|$ and $d_{L_i}v_i(t)$ represent the hydrodynamic drag, and b_i is the buoyancy. Using lumped parameters, (4) can be written as:

$$\dot{v}_i(t) = \alpha_i\tau_i + \beta_i v_i(t)|v_i(t)| + \mu_i v_i(t) + \nu_i. \quad (5)$$

The decoupled single degree of freedom, lumped parameter, dynamical plant model can be written in vector form as

$$\dot{v}(t)_i = \Phi_i^T f(t)_i, \quad (6)$$

where $\Phi_i = [\alpha_i; \beta_i; \mu_i; \nu_i]_{4\times1}$ is the vector of lumped plant parameters and $f(t)_i = [\tau(t)_i; v_i(t)|v_i(t)|; v_i(t); 1]_{4\times1}$ is a nonlinear vector of state and the control input $\tau(t)_i$. The lumped parameters are defined in Table 2.

Lumped Parameters	Physical Definition
α_i	m_i^{-1}
β_i	$-m_i^{-1}d_{Q_i}$
μ_i	$-m_i^{-1}d_{L_i}$
ν_i	$-m_i^{-1}b_i$

Table 2: Lumped Parameters

1.2 Review of Dynamic Positioning of Underwater Vehicles

Most dynamically positioned marine vehicles in use today employ Proportional-Derivative (PD) or Proportional-Integral-Derivative (PID) controllers for each controlled degree-of-freedom of the form

$$\tau_d \quad = K_p\Delta x_{body} + K_d\Delta v + K_i\int_0^t \Delta x_{body}(\tau)d\tau \quad (7)$$

where Δx_{body} and Δv are the position and velocity tracking errors, respectively, in body coordinates, and K_p, K_i, and K_d are empirically tuned feedback gain matrices, and τ_d is the vector of net forces and moments commanded to the vehicle thruster system.

The development of model-based control techniques for the dynamic positioning of underwater robotic vehicles has been limited by the paucity of *experimentally validated* plant models. Although a variety of authors have reported the development dynamical models for underwater vehicles, very few report direct experimental validation of their reported plant dynamics models for low-speed maneuvering. Most control studies demonstrate the performance of the resulting closed-loop system in computer simulations alone, e.g. [20, 10, 40, 2], or employ non-parametric control methodologies that do not require knowledge of the plant dynamics [7].

Several reported studies address the special case of propulsion-less and appendage-less symmetric vehicles in inviscid fluid for which the PDE fluid component has a closed form solution, e.g. [31]. Relatively few studies, e.g. [23, 43], have reported experiments which experimentally identify the plant parameters for dynamical plant models in multiple degrees of freedom; even fewer studies, [33, 6, 38], report both experimentally determined model parameters *and* compare experimentally observed vehicle dynamical response with that predicted by the proposed analytical models.

2 Decoupled Single Degree of Freedom Dynamical Plant Model for the JHU ROV

In this section we present experimentally identified, decoupled, single degree of freedom dynamical plant models of the form (4), for the Johns Hopkins University Remotely Operated Vehicle (JHU ROV). A set of plant model parameters was identified for the x_1, x_2, x_3 , and x_4 degrees of freedom using a stable adaptive identifier, originally reported in [38]. The scalar adaptive identifier was run on the experimental data collected by conducting dynamic vehicle trials in which the JHU ROV was commanded to follow an open loop sinusoidal thrust profile. The output of this process is an estimated plant velocity, $\hat{v}(t)_i$, as well as estimates for the unknown plant parameters.

How well does the identified plant model performance agree with experimentally observed actual vehicle motion? To answer this question we ran simulations on the adaptively identified plant parameters, in each degree of freedom *i*. The output of the simulations was a velocity, v_{model_i}. The logged experimental plant velocity, v_{p_i}, was then compared to the velocity predicted by the dynamical plant model, v_{model_i}. The error for each degree of freedom *i*, is calculated as $e_i = mean(|v_{model_i} - v_{p_i}|)$. The best identified parameters were those with the lowest "error", and are listed in Table 3.

As seen by the error listed in Table 3, the identified dynamical plant models were able to predict the dynamical behavior of the JHU ROV down to $1.6\, cm/s$ in the x_1, x_2, and x_3 DOF, and $1.651\, degrees/s$ in the x_4 DOF.

3 Model Based Control

In this section we present five different model based controllers and a basic Proportional-Derivative (PD) controller. The form of each controller is presented followed by a stability analysis of the resulting closed loop dynamical system. The model based controllers were designed to track both position and velocity trajectories.

3.1 PD Controller

The basic PD controller takes the form

$$\begin{aligned} \tau(t) &= k_p \Delta x(t) + k_d \Delta v(t), \\ & \quad k_p, k_d > 0, \end{aligned} \tag{8}$$

where k_p and k_d are scalar error feedback gains. The state error coordinates are defined as

$$\begin{aligned} \Delta x(t) = x_d(t) - x(t), \\ \Delta v(t) = v_d(t) - v(t), \\ \Delta \dot{v}(t) = \dot{v}_d(t) - \dot{v}(t), \end{aligned} \tag{9}$$

where $\dot{v}_d(t)$, $v_d(t)$, and $x_d(t)$ are the desired acceleration, velocity, and position. Note that $v(t) = d(x(t))/dt$ and $v_d(t) = d(x_d(t))/dt$. Substituting (8) into (3) the resulting closed loop dynamical system is

$$\begin{aligned} m\dot{v}(t) + d_Q v(t)|v(t)| + d_L v(t) + b- \\ k_p \Delta x(t) - k_d \Delta v(t) = 0. \end{aligned} \tag{10}$$

In the case where the buoyancy term, b, is zero, the PD controller will perform setpoint regulation, but not trajectory tracking.

3.2 Exact Linearizing Model Based Controller

The Exact Linearizing Model Based Controller (EL) takes the form

$$\begin{aligned} \tau(t) &= m\dot{v}_d(t) + d_Q v(t)|v(t)| + \\ & \quad d_L v_d(t) + b + k_p \Delta x(t) + k_d \Delta v(t), \\ & \quad m, k_p, k_d > 0,\ d_Q, d_L \geq 0, \end{aligned} \tag{11}$$

where m, d_Q, d_L, and b are known scalar quantities, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The state error coordinates are defined in (9). Substituting (11) into (3) the resulting closed loop dynamical system is

$$m\Delta\dot{v}(t) + (d_L + k_d)\Delta v(t) + k_p \Delta x(t) = 0, \tag{12}$$

a time varying linear system. Consider the Lyapunov function candidate

$$\begin{aligned} W(\Delta v(t), \Delta x(t)) = \tfrac{m^{-1}k_p+1}{2m^{-1}(d_L+k_d)}\Delta x(t)^2 + \\ mk_p^{-1}\Delta x(t)\Delta v(t) + \tfrac{mk_p^{-1}+1}{2m^{-1}(d_L+k_d)}\Delta v(t)^2. \end{aligned} \tag{13}$$

Degree of Freedom	α_i	β_i	μ_i	ν_i	Error e_i
1 (x_1)	0.003755	-4.406	0	0.007358	0.01664 m/s
1 (x_2)	0.002355	-4.9952	0	-0.004066	0.01607 m/s
3 (x_3)	6.237e-4	-1.102	0	0.01934	0.01652 m/s
4 (x_4)	0.0102	-1.9086	0	-0.001221	1.651 degrees/s
Degree of Freedom	Inertia	D_Q	D_L	Buoyancy	
1 (x_1)	266.3 [kg]	1173.4 [kg/m]	0 [kg/s]	-1.96 [N]	
2 (x_2)	424.6 [kg]	2120.9 [kg/m]	0 [kg/s]	1.73 [N]	
3 (x_3)	1603.3 [kg]	1766.84 [kg/m]	0 [kg/s]	-31.01 [N]	
4 (x_4)	98.04 [kg m^2]	187.1 [kg m^2]	0 [(kg m^2)/s]	0.1197 [N-m]	

Table 3: Adaptive ID Lumped Parameters

The time derivative of (13),

$$\dot{W}(\Delta v(t), \Delta x(t)) = -(\Delta x(t)^2 + \Delta v(t)^2), \quad (14)$$

is negative definite. Therefore the closed loop dynamical system (12) has a globally asymptotically stable equilibrium point at the origin $(\Delta x(t), \Delta v(t) = 0)$. We conclude that $\lim_{t\to\infty} \Delta x(t) = 0$ and $\lim_{t\to\infty} \Delta v(t) = 0$.

3.3 Non-Linear Model Based Controller

The Non-Linear Model Based Controller (NL) takes the form

$$\begin{aligned}\tau(t) &= m\dot{v}_d(t) + d_Q v_d(t)|v_d(t)| + d_L v_d(t) + \\ &\quad b + k_p \Delta x(t) + k_d \Delta v(t), \\ &\quad m, k_p, k_d > 0,\ d_Q, d_L \geq 0, \quad (15)\end{aligned}$$

where m, d_Q, d_L, and b are known scalar quantities, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The state error coordinates are defined in (9). Substituting (15) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}&m\Delta\dot{v}(t) + d_q(v_d(t)|v_d(t)| - v(t)|v(t)|) + \\ &\quad (d_L + k_d)\Delta v(t) + k_p \Delta x(t) = 0. \quad (16)\end{aligned}$$

or rewritten as

$$\begin{aligned}&\Delta\dot{v}(t) = -m^{-1}(d_q(v_d(t)|v_d(t)| - v(t)|v(t)|) + \\ &\quad (d_L + k_d)\Delta v(t) + k_p \Delta x(t)). \quad (17)\end{aligned}$$

Consider the Lyapunov function candidate

$$W(\Delta v(t), \Delta x(t)) = 0.5 k_p \Delta x(t)^2 + 0.5\, m \Delta v(t)^2. \quad (18)$$

The time derivative of (18) is

$$\begin{aligned}&\dot{W}(\Delta v(t), \Delta x(t)) = -(d_L + k_d)\Delta v(t)^2 - \\ &\quad d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t). \quad (19)\end{aligned}$$

Examining the term $(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t)$ we see that when $\Delta v(t) > 0$ we have $v(t) < v_d(t)$ which results in $(v_d(t)|v_d(t)| - v(t)|v(t)|) > 0$ and $(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t) > 0$. Likewise if $\Delta v(t) < 0$ then we have $v(t) > v_d(t)$, resulting in $(v_d(t)|v_d(t)| - v(t)|v(t)|) < 0$ and $(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t) > 0$. Thus (19) is negative semi-definite. The resulting closed loop system, (16), has an equilibrium point at the origin $(\Delta x(t), \Delta v(t) = 0)$ that is globally stable. Therefore, both $\Delta x(t)$ and $\Delta v(t)$ are bounded. We conclude the following

1. We conclude that $\Delta x(t)$ and $\Delta v(t)$ are bounded.
2. Since we define $v_d(t)$ and $x_d(t)$ to be bounded signals, as a result of 1 we conclude that both $v(t)$ and $x(t)$ are bounded.
3. As a consequence of 2 and $m > 0$, we get from (17) that $\Delta\dot{v}(t)$ is bounded.
4. Given (19) and that $d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t) \geq 0$ as argued earlier, we get $\Delta v(t)^2 \leq -\frac{W(\Delta v(t), \Delta x(t))}{(d_L + k_d)}$.
5. As a consequence of 2 and 3, we claim that $\Delta v(t) \in L^2$ and conclude that $\lim_{t\to\infty} \Delta v(t) = 0$.

In summary we have that the position tracking error is globally stable and the velocity error globally, asymptotically tracks to zero.

3.4 Adaptive Exact Linearizing Model Based Controller

The Adaptive Exact Linearizing Model Based Controller (AEL) takes the form

$$\begin{aligned}\tau(t) &= \hat{m}(t)\dot{v}_d(t) + \hat{d}_Q(t) v(t)|v(t)| + \\ &\quad \hat{d}_L(t) v_d(t) + \hat{b}(t) + k_p \Delta x(t) + k_d \Delta v(t), \\ &\quad k_p, k_d > 0, \quad (20)\end{aligned}$$

where $\hat{m}(t)$, $\hat{d}_Q(t)$, $\hat{d}_L(t)$, and $\hat{b}(t)$ are adaptive estimates of the scalar plant parameter values, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The plant parameter error coordinates are defined as

$$\begin{aligned}\Delta m(t) &= \hat{m}(t) - m,\\ \Delta d_Q(t) &= \hat{d}_Q(t) - d_Q,\\ \Delta d_L(t) &= \hat{d}_L(t) - d_L,\\ \Delta b(t) &= \hat{b}(t) - b,\\ \Delta\Psi(t) &= \hat{\Psi}(t) - \Psi,\\ \Delta\dot{\Psi}(t) &= \dot{\hat{\Psi}}(t);\ \dot{\Psi} = 0, \end{aligned} \quad (21)$$

where $\hat{\Psi}(t) = [\hat{m}(t); \hat{d}_Q(t); \hat{d}_L(t); \hat{b}(t)]_{4\times1}$ is an estimate of the unknown plant parameter vector $\Psi = [m; d_Q; d_L; b]_{4\times1}$. Substituting (20) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}\Delta m(t)\dot{v}_d(t) + \Delta d_Q(t)v(t)|v(t)| + \Delta d_L(t)v_d(t) +\\ \Delta b(t) + m\Delta\dot{v}(t) + d_L\Delta v(t) +\\ k_p\Delta x(t) + k_d\Delta v(t) = 0,\end{aligned} \quad (22)$$

or rewritten as

$$\begin{aligned}\Delta\dot{v}(t) = -m^{-1}(\Delta\Psi(t)^T X(t) +\\ (d_L + k_d)\Delta v(t) + k_p\Delta x(t)),\end{aligned} \quad (23)$$

where $X(t) = [\dot{v}_d(t); v(t)|v(t)|; v_d(t); 1]_{4\times1}$. Note that if the parameter error $\Delta\Psi(t)$, were zero, then (22) becomes the EL closed loop dynamical system (12).

Define the Lyapunov function candidate as

$$\begin{aligned}W(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = 0.5k_p\Delta x(t)^2 +\\ 0.5m\Delta v(t)^2 + 0.5\Delta\Psi(t)^T\Gamma^{-1}\Delta\Psi(t)\end{aligned} \quad (24)$$

We can design (24) to have a negative semi-definite time derivative by choosing the parameter update law to be

$$\dot{\hat{\Psi}}(t)_{4\times1} = \Gamma X(t)\Delta v(t), \quad (25)$$

where $\Gamma_{4\times4} = diag[\gamma_i]$ and $\gamma_i \geq 0$ for i$= 1\cdots4$. The time derivative of (24) is then

$$\dot{W}(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = -(k_d + d_L)\Delta v(t)^2. \quad (26)$$

We conclude the following

1. From (24) and (26), we conclude that $\Delta x(t)$, $\Delta v(t)$ and $\Delta\Psi(t)$ are bounded.
2. Given that $\Delta\Psi(t)$ is bounded and Ψ is a constant, then $\hat{\Psi}(t)$ is bounded as well.
3. If we define $x_d(t)$, $v_d(t)$, and $\dot{v}_d(t)$ to be bounded, then from (21) we conclude that x(t) and v(t) are bounded.
4. From (23), 1, 2, 3, and given that $m \neq 0$, k_p, and k_d are constant we declare $\Delta\dot{v}(t)$ to be bounded.
5. In consequence, given 3, $\dot{v}(t)$ is also bounded.
6. From (26) and that $\Delta x(t)$, $\Delta v(t)$ and $\Delta\Psi(t)$ are bounded, we can then conclude that $\Delta v(t) \in L^2$, and given that $\Delta\dot{v}(t)$ is bounded, we can conclude $\lim_{t\to\infty}\Delta v(t) = 0$.
7. Given that $\lim_{t\to\infty}\Delta v(t) = 0$, Γ is constant, and $X(t)$ is bounded, then from (25) we conclude $\lim_{t\to\infty}\Delta\dot{\Psi}(t) = 0$.

In summary we have concluded that all signals remain bounded, the velocity tracking error globally, asymptotically goes to zero (the time derivative of the position error asymptotically goes to zero), and the time derivative of the parameter estimates converges asymptotically to zero. However, absent additional arguments we cannot claim either $\lim_{t\to\infty}\Delta\Psi(t) = 0$ or that $\lim_{t\to\infty}\Delta x(t) = 0$.

3.5 Adaptive Non-Linear Model Based Controller

In this section the Adaptive Non-Linear Model Based Controller is presented. The ANL takes the form

$$\begin{aligned}\tau(t) &= \hat{m}(t)\dot{v}_d(t) + \hat{d}_Q(t)v_d(t)|v_d(t)| +\\ &\quad \hat{d}_L(t)v_d(t) + \hat{b}(t) + k_p\Delta x(t) + k_d\Delta v(t),\\ &\quad k_p, k_d > 0,\end{aligned} \quad (27)$$

where $\hat{m}(t)$, $\hat{d}_Q(t)$, $\hat{d}_L(t)$, and $\hat{b}(t)$ are estimates of the scalar plant parameter values, $\tau(t)$ is the controller output, and k_p and k_d are PD scalar error feedback gains. The plant parameter error coordinates are defined in (21) and the state error coordinates are defined in (9). Substituting (27) into (3) the resulting closed loop dynamical system is

$$\begin{aligned}\Delta m(t)\dot{v}_d(t) + \Delta d_Q(t)v_d(t)|v_d(t)| +\\ \Delta d_L(t)v_d(t) + \Delta b(t) + m\Delta\dot{v}(t) +\\ d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|) + d_L\Delta v(t) +\\ k_p\Delta x(t) + k_d\Delta v(t) = 0,\end{aligned} \quad (28)$$

which can be rewritten as

$$\begin{aligned}\Delta\dot{v}(t) = -m^{-1}(\Delta\Psi(t)^T X(t) +\\ d_q(v_d(t)|v_d(t)| - v(t)|v(t)|) +\\ (d_L + k_d)\Delta v(t) + k_p\Delta x(t)),\end{aligned} \quad (29)$$

where $X(t) = [\dot{v}_d(t); v_d(t)|v_d(t)|; v_d(t); 1]_{4\times1}$. Note that if the parameter error were zero, $\Delta\Psi(t) = 0$, then (28) becomes the NL closed loop dynamical system (16).

Consider the Lyapunov function candidate

$$W(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = 0.5k_p\Delta x(t)^2 + 0.5m\Delta v(t)^2 + 0.5\Delta\Psi(t)^T\Gamma^{-1}\Delta\Psi(t). \quad (30)$$

We can design (30) to have a negative semi-definite time derivative by choosing the parameter update law to be

$$\dot{\hat{\Psi}}(t)_{4\times1} = \Gamma X(t)\Delta v(t), \quad (31)$$

where $\Gamma_{4\times4} = diag[\gamma_i]$ and $\gamma_i \geq 0$ for i= $1\cdots4$. The resulting time derivative of (24) is

$$\dot{W}(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = -(k_d + d_L)\Delta v(t)^2 - d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t). \quad (32)$$

The first term in (32) is negative definite in $\Delta v(t)$, and following the same argument made in Section 3.3, we can conclude that (32) is negative semi-definite.

We conclude the following,

1. From (30) and (32), we conclude that $\Delta x(t)$, $\Delta v(t)$ and $\Delta\Psi(t)$ are bounded.

2. Given that $\Delta\Psi(t)$ is bounded and Ψ is a constant, then $\hat{\Psi}(t)$ is bounded as well.

3. If we define $x_d(t)$, $v_d(t)$, and $\dot{v}_d(t)$ to be bounded, then x(t) and v(t) are bounded.

4. From (29), 1, 2, 3, and given that $m \neq 0$, k_p, k_d are constant we declare $\Delta\dot{v}(t)$ to be bounded.

5. In consequence, given 3, $\dot{v}(t)$ is also bounded.

6. From (32) and that $d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t) \geq 0$ as argued earlier, we get $\Delta v(t)^2 \leq -\frac{W(\Delta v(t), \Delta x(t))}{(d_L + k_d)}$.

7. As a result of 1 and 6, we claim that $\Delta v(t) \in L^2$ and coupled with 5 we conclude that $\lim_{t\to\infty} \Delta v(t) = 0$.

8. Given 7, Γ is constant, and $X(t)$ is bounded, then from (31) we conclude $\lim_{t\to\infty} \Delta\dot{\Psi}(t) = 0$.

In summary we have concluded that all signals remain bounded, that both the velocity tracking error is globally asymptotically stable, and the time derivative of the parameter estimates converge asymptotically to zero. However, absent additional arguments we cannot claim either $\lim_{t\to\infty} \Delta\Psi(t) = 0$ or that $\lim_{t\to\infty} \Delta x(t) = 0$.

3.6 Epsilon Adaptive Model Based Controller

Although the ANL controller was shown to be stable, bounded, and to provide asymptotically exact velocity tracking, it does not guarantee asymptotically exact position tracking. This apparent defect can be corrected with a slightly modification of the parameter update law. The idea is to introduce a small cross term of the form $\epsilon\Delta x(t)\Delta v(t)$, $\epsilon > 0$, to the Lyapunov function. If ϵ is sufficiently small, the Lyapunov function remains positive definite and radially unbounded and its time derivative is locally negative definite. This local stability result was first reported independently in [4, 28, 41]. A globally stable variation was reported in [42]. Applying this approach to the present plant, (3), the control law (27) remains unchanged, resulting in the closed loop dynamical system (28).

Consider the Lyapunov function candidate

$$W(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = 0.5k_p\Delta x(t)^2 + 0.5m\Delta v(t)^2 + \epsilon\, m\Delta x(t)\Delta v(t) + 0.5\Delta\Psi(t)^T\Gamma^{-1}\Delta\Psi(t). \quad (33)$$

For ϵ sufficiently small, this function is positive definite and radially unbounded. Choosing the new parameter update law

$$\dot{\hat{\Psi}}(t) = \Gamma X(t)(\Delta v(t) + \epsilon\Delta x(t)), \quad (34)$$

where $X(t) = [\dot{v}_d(t), v_d(t)|v_d(t)|, v_d(t), 1]_{4\times1}$, $\epsilon > 0$, $\Gamma_{4\times4} = diag[\gamma_i]$, $\gamma_i \geq 0$ for $i = 1\cdots4$, the time derivative of (33) is

$$\begin{aligned}\dot{W}(\Delta x(t), \Delta v(t), \Delta\Psi(t)) = \\ -(k_d + d_L - \epsilon m)\Delta v(t)^2 - \\ d_Q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta v(t) - \\ \epsilon k_p\Delta x(t)^2 - \epsilon(k_d + d_L)\Delta x(t)\Delta v(t) - \\ \epsilon d_q(v_d(t)|v_d(t)| - v(t)|v(t)|)\Delta x(t). \quad (35)\end{aligned}$$

As reported in [4, 28, 41, 42], the time derivative of this Lyapunov function is locally negative definite. Note that in the case of $\epsilon = 0$, the ϵANL is identical to the ANL controller.

We conclude that the ϵANL controller is locally stable.

4 Simulated Performance of Controllers

An advantage of having an experimentally validated dynamical plant model of the JHU ROV is that we can simulate the performance of the model

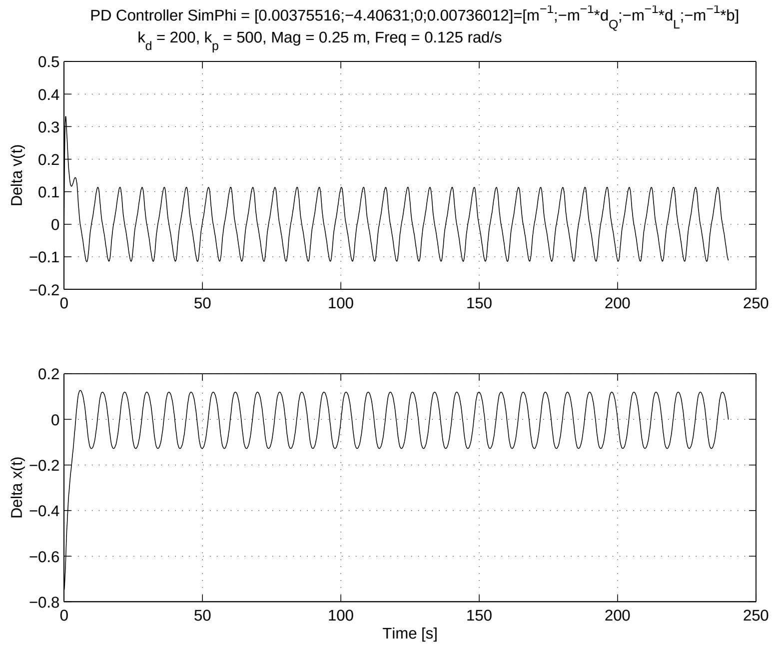

Figure 1: Simulation of PD controller in x_1 DOF. Plot shows velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

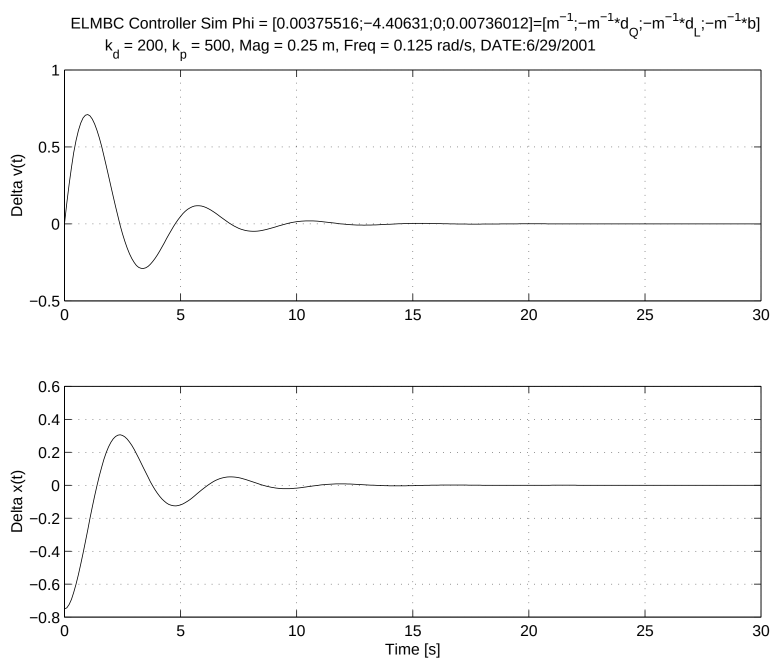

Figure 2: Simulation of EL controller in x_1 DOF. Plot shows velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

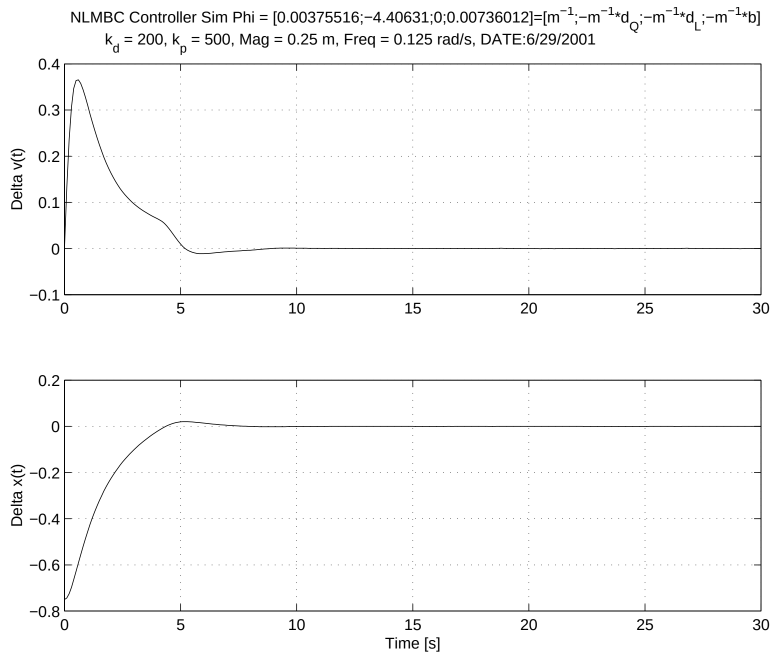

Figure 3: Simulation of NL controller in x_1 DOF. Plot shows velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

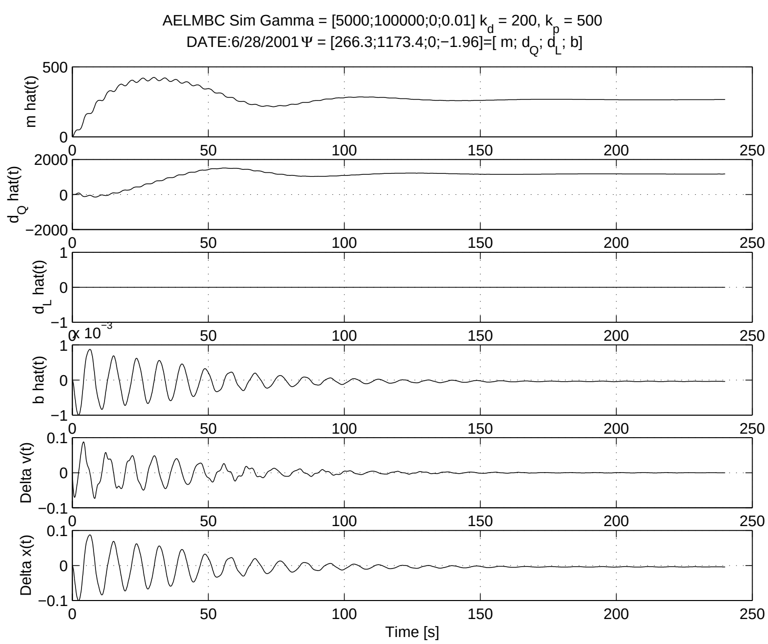

Figure 4: Simulation of AEL controller in x_1 DOF. Plot shows adaptive parameter estimates $(\hat{m}(t), \hat{d_Q}(t), \hat{d_L}(t), \hat{b}(t))$, velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

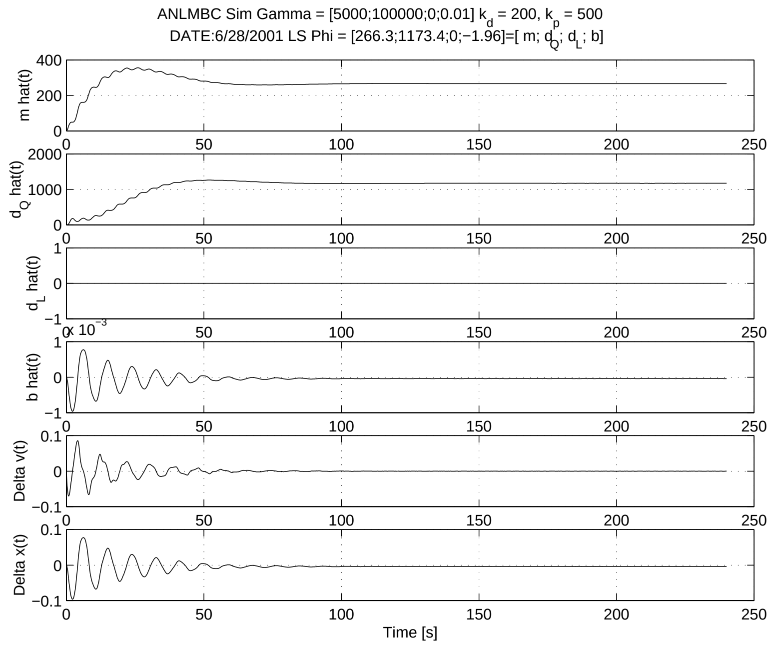

Figure 5: Simulation of ANL controller in x_1 DOF. Plot shows adaptive parameter estimates $(\hat{m}(t), \hat{d_Q}(t), \hat{d_L}(t), \hat{b}(t))$, velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

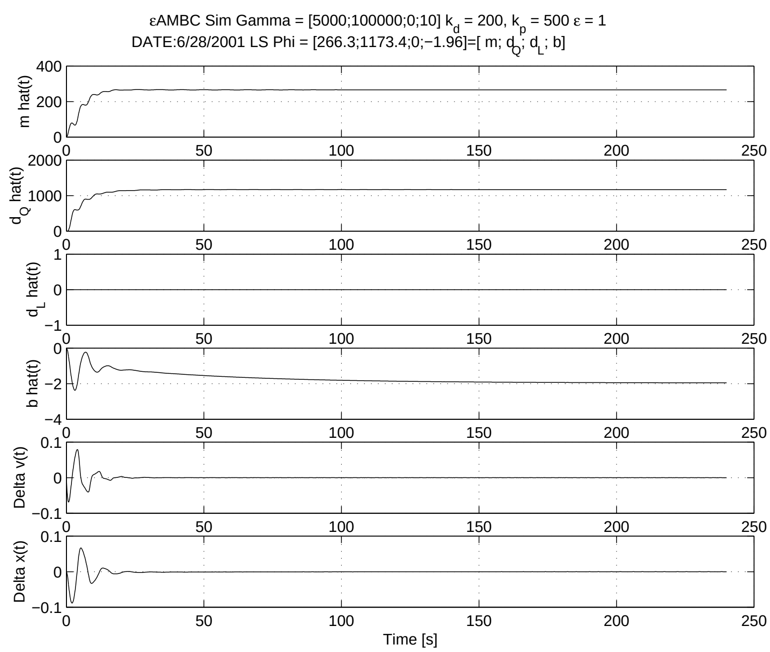

Figure 6: Simulation of ϵANL controller in x_1 DOF. Plot shows adaptive parameter estimates $(\hat{m}(t), \hat{d_Q}(t), \hat{d_L}(t), \hat{b}(t))$, velocity tracking error, $\Delta v(t)$, and position tracking error, $\Delta x(t)$.

based controllers prior to implementing them experimentally. Simulations were run in the x_1, x_2, x_3, and x_4 degrees of freedom, using each of the controllers presented in Section 3. The model parameters presented in Section 2 were used to simulate the JHU ROV's dynamical behavior. We present the results from the x_1 degree of freedom here.

In the case of the PD controller in Figure 1, the simulations show that both the position and velocity tracking errors remain bounded. As predicted by theory, we see in Figure 2 that both the velocity and position tracking error go to zero for the EL controller. Similarly for the NL controller, Figure 3 supports the theory in that both error signals remain bounded and the velocity tracking error goes to zero. In addition to what was predicted by theory, we see that the position tracking error goes to zero as well.

Figures 4 and 5 demonstrate the simulated performance of the AEL and ANL controllers. In the case of the AEL controller, we see that all signals remain bounded, the velocity error goes to zero and the adaptive parameters converge to a value as expected. Similarly for the ANL controller, all signals remain bounded, the velocity error goes to zero and the adaptive parameters converge to a value as expected. The position tracking error does not go to zero for either of the AEL or ANL controllers in simulation. In looking at the simulation results for the ϵANL, we see that the adaptive parameters converge to a final value and the velocity tracking error goes to zero. In addition, the position tracking error goes to zero.

5 Conclusion

This paper has addressed the problem of model based trajectory tracking control for the low-speed maneuvering of fully actuated underwater vehicles. We considered the simple, but empirically validated, case of a decoupled dynamical plant model employing constant added mass, linear and quadratic drag, buoyancy, and thrust input. Two fixed model-based controllers were considered: an exactly linearizing controller (EL), and a nonlinear controller (NL) that does not exactly linearize. Both are shown analytically to provide asymptotically exact velocity tracking. Adaptive versions of both controllers are reported and shown analytically to be stable. Numerical simulations of the closed-loop performance of these systems corroborate the theoretical predictions. The fixed model-based controllers provide asymptotically exact tracking, outperforming the PD controller in all cases except setpoint regulation. The adaptive model-based controllers all provide asymptotically exact velocity tracking, and the parameter estimates remain bounded and converge to fixed values. The addition of a position error feedback term to the adaptation update law in the ϵANL adaptive controller provides for asymptotically exact position tracking.

The simulations do not reveal the robustness of the proposed control algorithms with respect to (a) sensor noise and inaccuracy; (b) thruster accuracy, dynamic response and saturation; or (c) unmodeled plant dynamics. We hope to shortly report on an experimental study of these controllers to carefully evaluate the performance of these controller on actual underwater vehicles.

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